Experimental validation of usual numerical models of cracked beams for damage detection approaches

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ABSTRACT

The performances of damage detection approaches are often tested by using numerical solutions of undamaged and damaged structures. However, the numerical results used for a preliminary analysis are useful as far as they can provide a good approach to real experimental results. This paper presents a collection of numerical results obtained from three different finite element models that are extensively used in the literature for the analysis of damage detection approaches, and compare them with experimental results. The paper considers beam, plate and solid models. Natural frequencies and mode shapes of the beams are analyzed. The results show that the results obtained with 1D models are not likely to be well related to a specific experimental test, whereas 2D and 3D models provide good results if the damage is a notch, but, the do not if it is a fatigue crack.

Keywords: Damage detection, Modal analysis, Numerical Model.

1. INTRODUCTION

In the last decades, a significant number of papers have addressed the damage detection in structures through many different perspectives. A promising and well established approach is vibration based methodology. It is based in the analysis of changes of natural frequencies and mode shapes induced by damage.

In order to reduce experimental costs, numerical simulations of undamaged and damaged structures are extensively used to avoid setting up real experiments when developing and testing new damage detection methods. However, it is obvious that numerical simulations always lack of some realistic phenomena. One of these phenomena is experimental noise, which is usually simulated in the end by adding some artificial noise to the numerical results. On the other hand, the numerical model is always an approximation to the real structure and it may not consider nonlinear effects, geometrical imperfections, etc. For these reasons, although numerical simulations can be useful at a first stage of a research, it is necessary to finally test any damage detection methodology with real experimental examples. However, since numerical results are considered as benchmark, they should be able to provide a good approach to realistic results.

This paper is focused on the analysis of several types of finite element models that can be used to obtain the natural frequencies and mode shapes of cracked beams and that have already been analyzed in

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previous papers from a numerical as well as from a experimental point of view. The beams are simple rectangular cross section specimens made of steel, as they are frequently used in lab research.

Three different finite element models are considered by using beam, plate and solid elements. The damage (crack) is simulated by modelling the actual geometry of the damaged beam, considering the damage as a loss of material of certain volume (a saw cut type of damage) or simply a disconnection between adjacent nodes.

The numerical results are compared to real experimental results reported in the literature. The paper analyzes an undamaged beam with two [1] and one ends fixed [2], an undamaged beam with an added mass and two ends fixed [1], a beam with a notch and one [3,4] and two ends fixed [5], a simply supported beam with a notch [6] and with an added mass [7], a cantilever beam [4] and a simply supported beam [8] with a fatigue crack.

2. FINITE ELEMENT MODELS

The three different finite element models of the beam are used to obtain natural frequencies and mode shapes assuming a linear behavior of the structure. They have been built using Ansys software. The Mechanical APDL environment have been used in order to obtain results in an easy and efficient way for different boundary conditions, different length and cross section of the beam, different position and size of damage, and different position and size of added nonstructural mass, etc. Fig.1 shows some schematic views of the different models.

When modeling a notch, it is considered by eliminating its corresponding volume. Thus, the geometry of the FE model exactly represents the actual geometry of the beam, including the notch. The presence of damage in a beam have been extensively considered in that way for experimental [7,9,10,11] as well as for numerical analysis [12,13,14].

The beam model considers a reduced section of the beam according to the notch. The BEAM188 element from Ansys element library has been chosen. The plane model is built with PLANE182 element from Ansys library. The mid plane (plane of symmetry) of the beam is modelled considering a plane stress approach. The profile of the notch is part of the boundary of the finite element mesh. For the solid model, the SOLID187 element from Ansys library is chosen.

3. VALIDATION OF MODELS

Table 1: *Natural frequencies for a fixed-fixed beam.*

Mode number	Frequencies(Hz)			
	Low [8]	Beam model	Plate model	Solid model
First	25’15	25’21	26’33	25’33
Second	69’52	69’48	72’55	69’61

3.1.2. Cantilever beam

The experimental results for this case are obtained from the work of Qian, Gu y Jiang [2]. The beam is 0,2m long, 1mm wide and 7,8mm high. The paper indicates that the beam is made of steel, but the mechanical properties are not provided. For the numerical simulations, the following properties are considered: E=210GPa, ν=0.3, ρ=7850kg/m3.

Table 2 shows the experimental natural frequencies are accurately approached by the three finite element models.

Table 2: *Natural frequencies for a cantilever beam.*

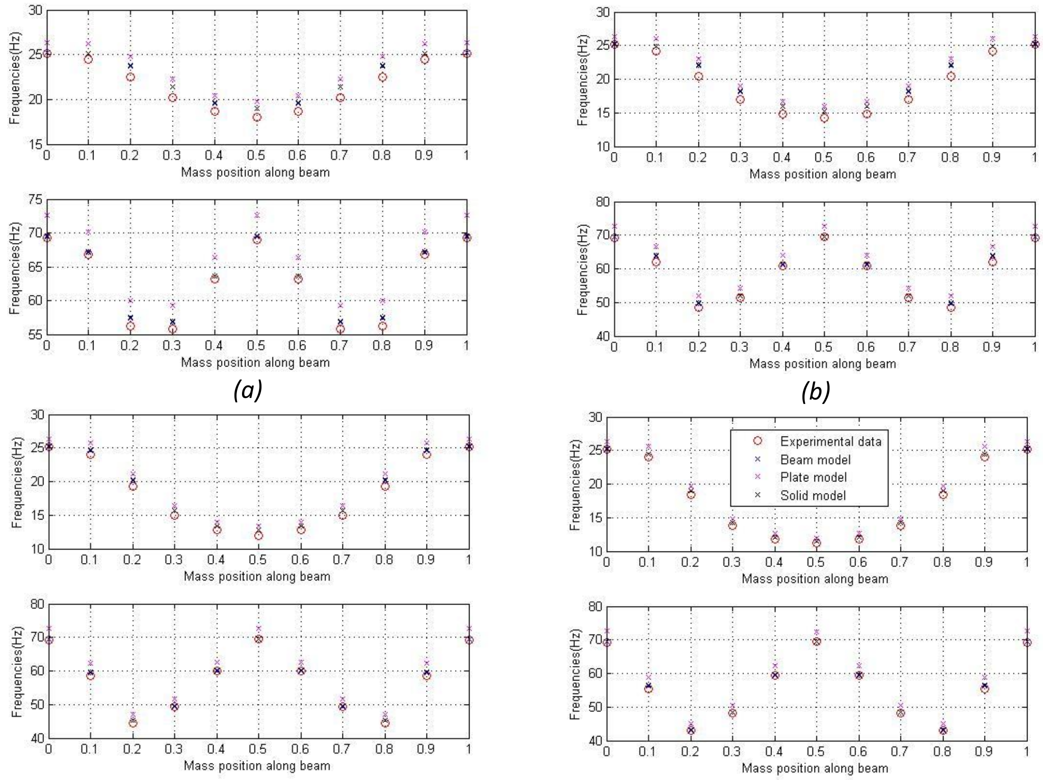
Mode number	Frequencies(Hz)			
	Qian et al. [9]	Beam model	Plate model	Solid model
First	162’65	162’73	163’76	162’85
Second	1019’24	1012’58	1019’14	1013’43
Third	2862’82	2803’60	2822’84	2806’46

3.2. Effect of nonstructural added mass

3.2.1. Fixed-fixed beam

The experimental results of this section were obtained by Low [1]. In that paper, the first two natural frequencies are obtained for different positions of the added mass along the beam, and different sizes of the mass are also considered. The properties of the beam are the same of that already presented in section 3.1.1 (1m long, 14,946mm wide, 4,765mm high, E=207GPa, ν=0.3, ρ=7810kg/m3).

Fig.2 compares the experimental and numerical results for all positions and sizes of the mass. It can be seen that the natural frequency changes according to the corresponding mode shape. More significant changes (drops) in natural frequency is obtained as the added mass approaches the areas of higher modal amplitudes.



(a) *(b)*

(c) *(d)*

Figure 2: *First two experimental and numerical natural frequencies for different positions of added mass along the beam and for different sizes of the mass: a) 0,30 b) 0,67 c) 1,12 and d)1,48 times the mass of the beam.*

The numerical results are in very good agreement with the experimental results for all cases. However, a better correlation is obtained for higher values of the added mass.

3.3. Beam with a notch

3.3.1. Cantilever beam

The results for this case are obtained from the work of Gudmunson [3,4]. Firstly [3], he analyzed the effect of a notch up to a depth of 40% the height of the beam. The beam was 0,2m long and 7,8mm high. The width is not specified in the paper. A value of 10mm has been used for the numerical models. The beam was made of steel. No specific values for mechanical properties were specified in the original paper. Standard values were considered for the finite element models (E=207GPa, ν=0.3, ρ=7810kg/m3).

The crack was artificially induced by a saw cut, which is the most usual way to induce damage in steel beams for damage detection analysis. The width of the notch was 0,7mm and the depth was 0mm (no damage), 1mm (12% of the height of the beam), 2mm (25%) and 3mm (40%). Several specimens were tested with different locations of the damage.

Fig. 3 shows the experimental results presented in [3] as well as the corresponding numerical values obtained with the three difference finite element models. The results are presented in terms of relative change in natural frequency (ratio between undamaged and damaged values) for the different sizes of the notch and different positions. The first three natural frequencies are analyzed when damage is at 0,025L (measured from the fixed end), the first and fourth values when damage is at 0,2L, and the fourth value when damage is at 0,5L.

The results presented in fig. 3 show that the beam model is not able to approach the experimental results. Thus, this type of model for which only some beam elements in the damaged area change their properties (section, stiffness, etc.) are not good for providing realistic results. A beam model may be used for testing numerically a damage detection approach but the results it provides cannot be directly related to a real damage. Since results are far from being good for this model, it is not going to be considered in the following sections.

The plate and solid models are in good agreement with experimental results. However, results are worst for increasing size of damage modes and higher modes.

Gudmunson [4] also obtained results for a notch of up to 90% the height of the beam. In that case, the beam was 0,3m long, 5mm wide, 25mm high and it was made of steel. The notch was also induced by a saw cut (0,4mm wide) and different depths were considered: 3, 6, 9, 12, 15, 18, 21 and 23mm. Damage was located at two different positions (0,1L and 0,3L). For all this damage scenarios, the second and third natural frequencies were obtained.

Fig. 4 shows the experimental and numerical results. It can be observed that both the plate and solid models provide accurate results even for such a severe damage in the beam. On the other hand, both models provide almost identical results.

3.3.2. Fixed-fixed beam

Hu et al [5] obtained the first three mode shapes for a beam with a double notch induced by a saw cut on top and bottom sides of the beam. The length of the beam was 0,6m, the width was 0,05m and the height was 0,006m. The mechanical properties of the beam were E=70GPa, ν=0.3 and ρ=2700kg/m3.

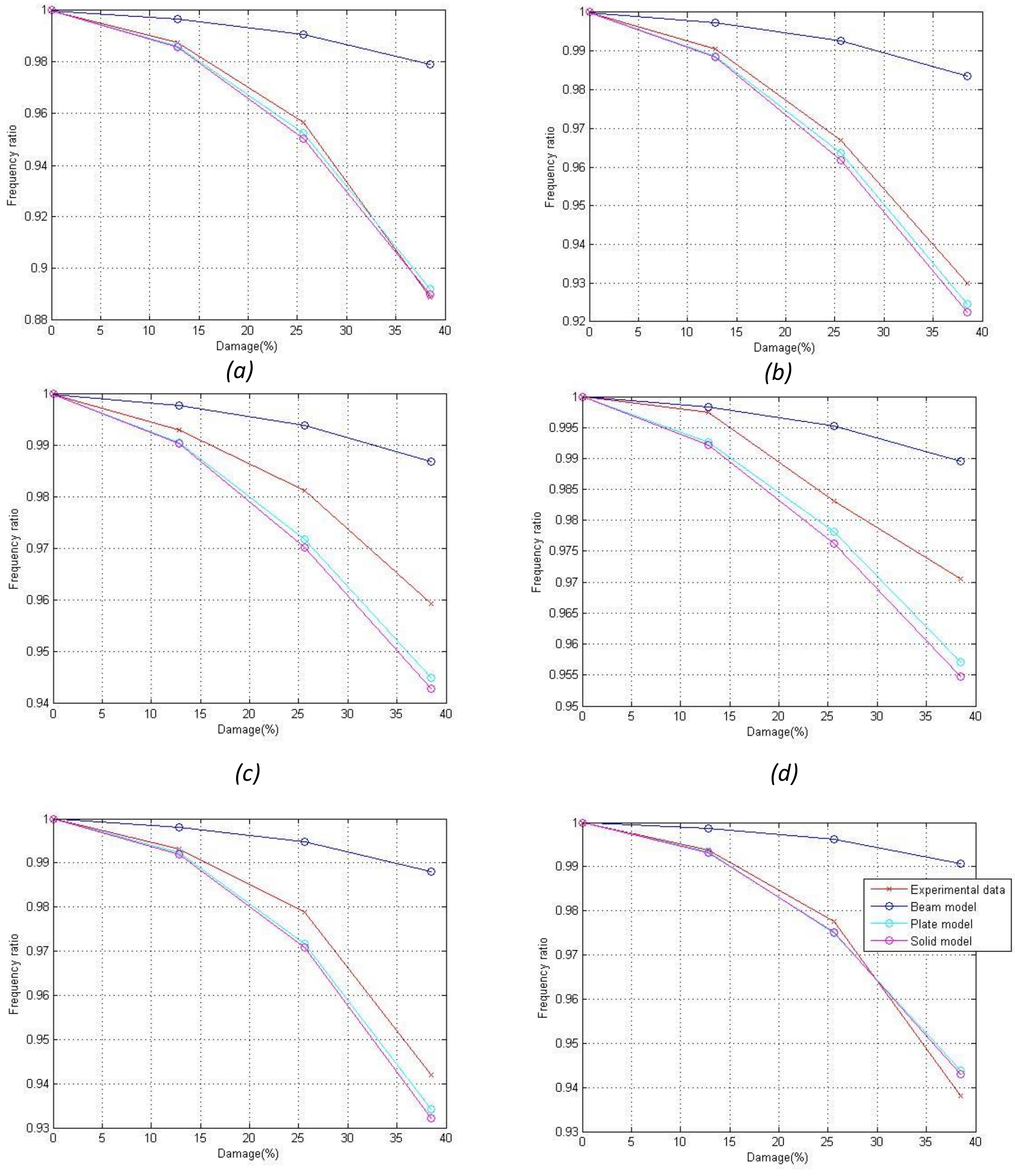


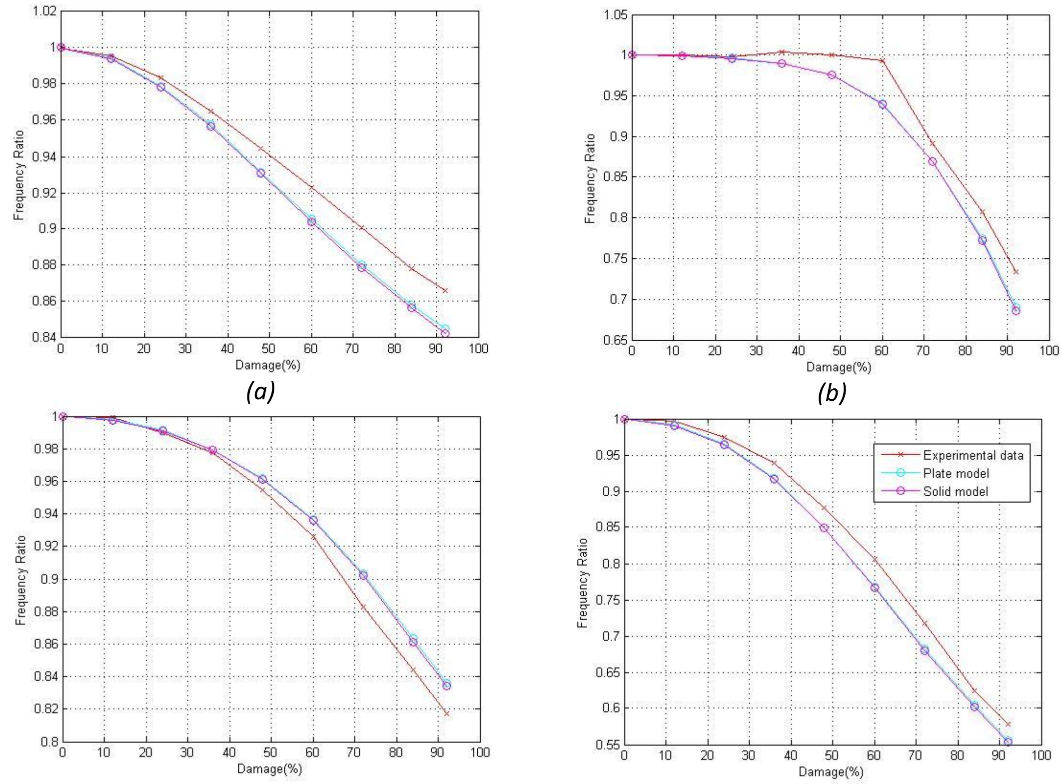
Figure 3: *Relative value of natural frequency for a cantilever beam with a notch at 0,025L (a, b, c for first, second and third values of natural frequency respectively), 0,2L (d, e for first and fourth values respectively) and 0,5L (f for fourth value).*

The notch was located at 0,255m from the left end. Fig.5 shows how the numerical mode shapes are in good agreement with the experimental mode shapes.

3.3.3. Simply supported beam

The experimental results for a simply supported beam has been obtained by Christides and Barr [6]. The beam was 0,575m long, 31,75mm high and 9,525mm wide. Its mechanical properties were E=210GPa, ν=0.3 and ρ=7850kg/m^3.

The beam has a double notch at the midpoint. Its width is not specified in the paper, so it is assumed a 0,5mm width for the numerical simulations. Fig. 6 shows the relative variation of the first natural



(a) *(b)*

(c) *(d)*

Figure 4: *Relative value of second (a, c) and third (b, d) natural frequency for a cantilever beam with a notch at 0,1L (a , b) and 0,3L (c, d).*

frequency for different depths of the notch. The numerical results for the solid model are in good agreement with the experimental ones.

3.3.4. Influence of the width of the notch

Silva y Gomes [15] analyzed the effect of the thickness of the notch. They obtained the natural frequencies for a free-free beam of rectangular cross section (0,016m x 0,032m) and 0,72m long, with a notch at 0,09m from one end of thickness from 0,2mm to 2mmm. The notches were induced precisely by a milling machine. The beam was made of steel (E=206GPa, ν=0.29 and ρ=7650kg/m3).

Table 3 shows the values of natural frequencies for different thickness of the notch. It can be observed that the thickness has very little influence on the natural frequencies of the beam. The experimental tests conducted by Silva and Gomes simulated free-free boundary conditions by means of a soft suspension at the ends of the beam. The stiffness of the suspension is not specified in the paper and the numerical model did not include it. The different boundary conditions may explain the difference between the experimental and numerical results. However, the trend is similar for both sets of results and it can be concluded that both the numerical model and the real beam are not sensitive to the width of the notch.

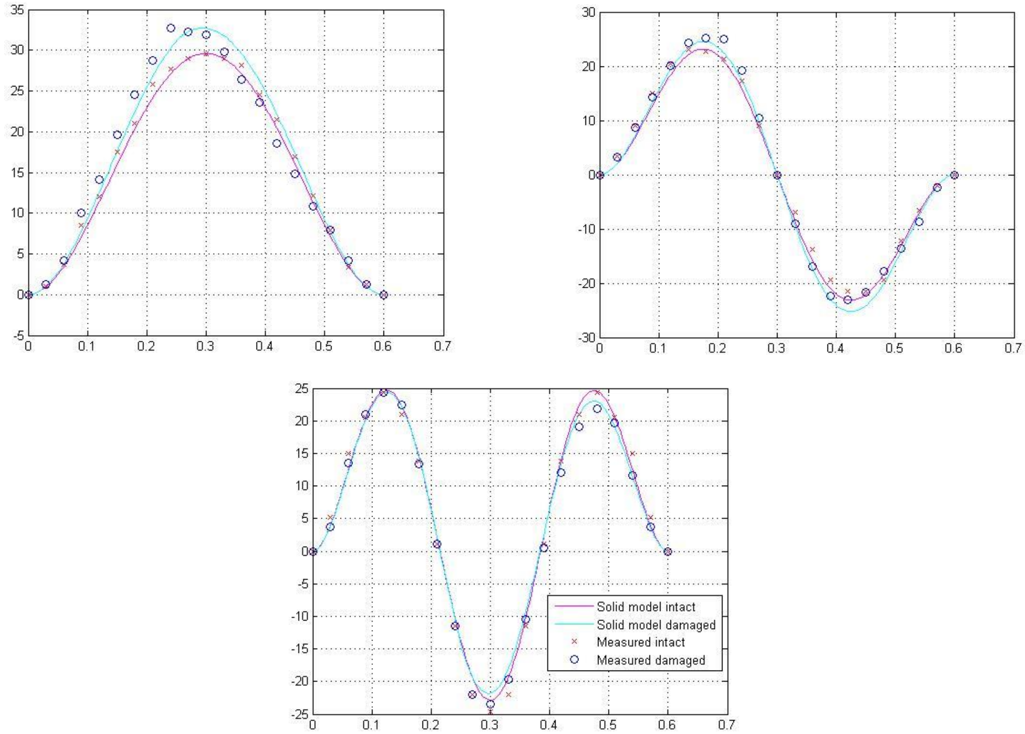


Figure 5: *Mode shapes for a fixed-fixed beam with a double notch at 0,255m*

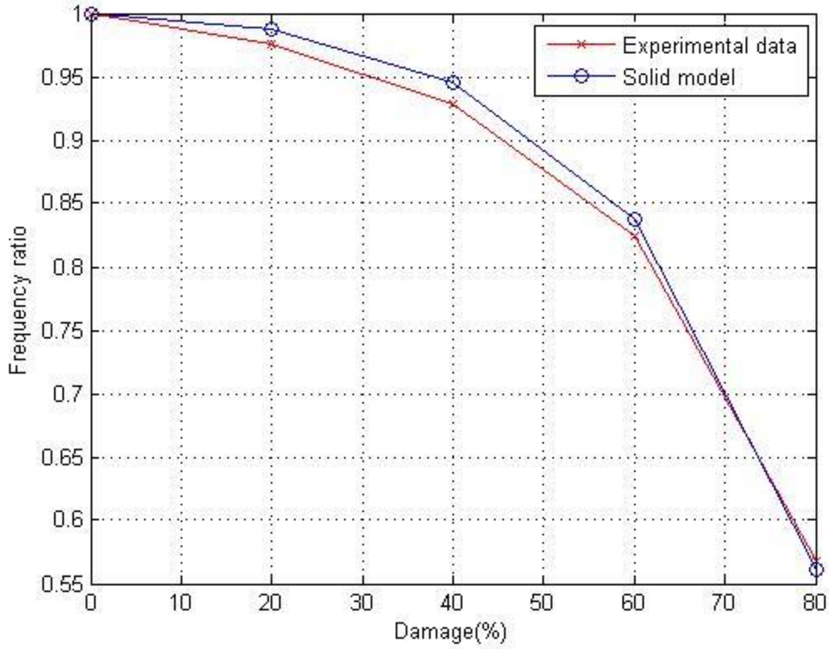


Figure 6: *relative variation of the first natural frequency for a simply supported beam.*

Table 3: *Influence of the thickness of the notch in the natural frequencies of a free-free beam.*

Mode number	Case	Frequencies (Hz)				
		Notch 2mm	Notch 1'5mm	Notch 1mm	Notch 0'5 mm	Notch 0'2 mm
First	FEM	0'28	0'29	0'29	0'29	0'29
	Silva[15]	0'65	0'65	0'65	0'65	0'65
Second	FEM	1'79	1'76	1'71	1'63	1'60
	Silva[15]	4'20	4'05	4'00	3'90	3'90
Third	FEM	4'44	4'37	4'22	4'10	4'02
	Silva[15]	9'80	9'25	9'05	8'95	8'90
Fourth	FEM	6'52	6'45	6'27	6'13	6'05
	Silva[15]	11'85	11'75	11'65	11'65	11'65

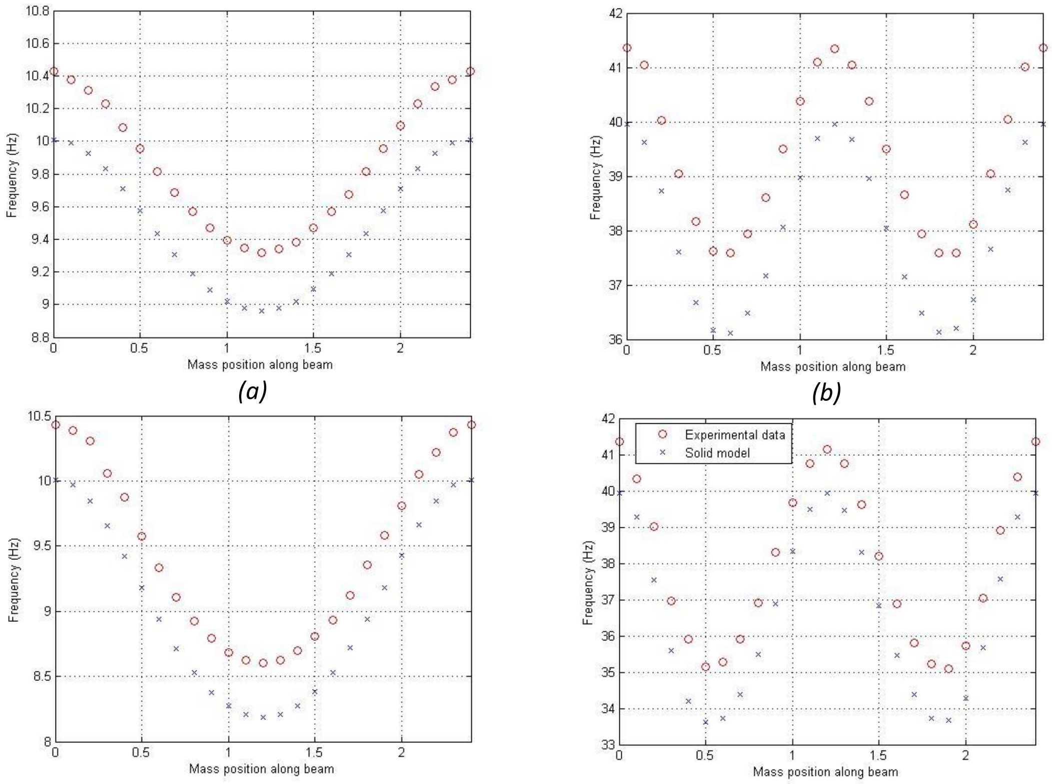
3.4. Added mass and notch

3.4.1. Simply supported beam

The results for this case has been obtained by Zhong and Oyadiji [7], they analyzed a 2,4m long beam made of aluminum, with a rectangular cross section of 0,025m x 0,1m. The mechanical properties for the numerical model were assumed to be E=70GPa, ν=0.33 and ρ=2700kg/m3.

The notch was induced with a saw cut at 0,4m from one end. Its depth was 5mm. The thickness was not specified in the paper, so it is assumed a value of 1mm for the numerical simulations. The added mass was located at different positions along the beam in order to see its effect on the natural frequencies of the beam. Two different values of the mass were considered, 2 and 4kg, corresponding to 12,3 and 25% of the mass of the beam respectively.

Fig. 7 shows that the numerical solid model predicts accurately the experimental results. A constant



(c) (d)

Figure 7: *First (a,c) and second (b,d) natural frequencies for a simply supported beam with a notch and 2 kg (a,b) or 4 kg (c,d) added mass.*

drift is present between the two sets of results. It may be due to some inaccurate assumptions in the numerical model (namely the mechanical properties of the material).

3.5. Crack

3.5.1. Cantilever beam

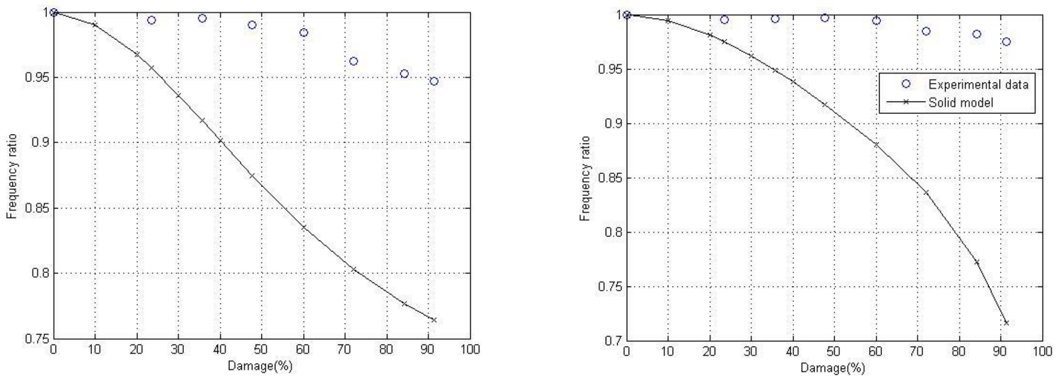
Gudmunson [4], studied the effect of a fatigue crack in a beam (0,3m long, 5mm width and 25mm high). For the numerical simulations, it was assumed that the beam was made of steel (E=210GPa, ν=0.3 and ρ=7850kg/m3).

The crack was induced by vibrating the beam with a small notch at 15mm from the fixed end. It is assumed a 1mm wide crack in the numerical model. Fig. 8 shows that the effect of the crack on the second and third natural frequencies of the beam is much more significant in the numerical model than in the experimental setup. Thus, the model is not able to provide realistic values for this type of crack.

3.5.2. Simply supported beam

Dimarogonas, Yao and Chondros [8] obtained results for a simply supported beam (235mm long, 23 mm wide and 7mm high) made of aluminum (E=72GPa, ν=0.35 and ρ=2800kg/m3). The crack was induced at the mid-section of the beam. A small notch was induced and then a fatigue crack was induced by vibrating the beam at its first natural frequency. The first natural frequency was obtained for different sizes (depth) of the crack. Two different type of tests were carried out, in order to analyze the effect of the breathing of the crack. In the first one, the breathing of the crack takes place so nonlinear effects may be present. In the second one, a small mass is added in order to keep the crack open. The added mass makes the natural frequency to decrease 1,5% approximately.

As in the previous section, fig. 9 shows that, since the numerical model is not considering nonlinear effects and no contact forces takes places at the faces of the crack, the numerical results are more influenced by the presence of the crack than it really happens experimentally. When the crack is forced to be open in the tests, then the numerical results are in better agreement to the experimental results.



(a) *(b)*

Figure 8: *Relative value of second (a) and third (b) natural frequencies of a cantilever beam with a fatigue crack.*

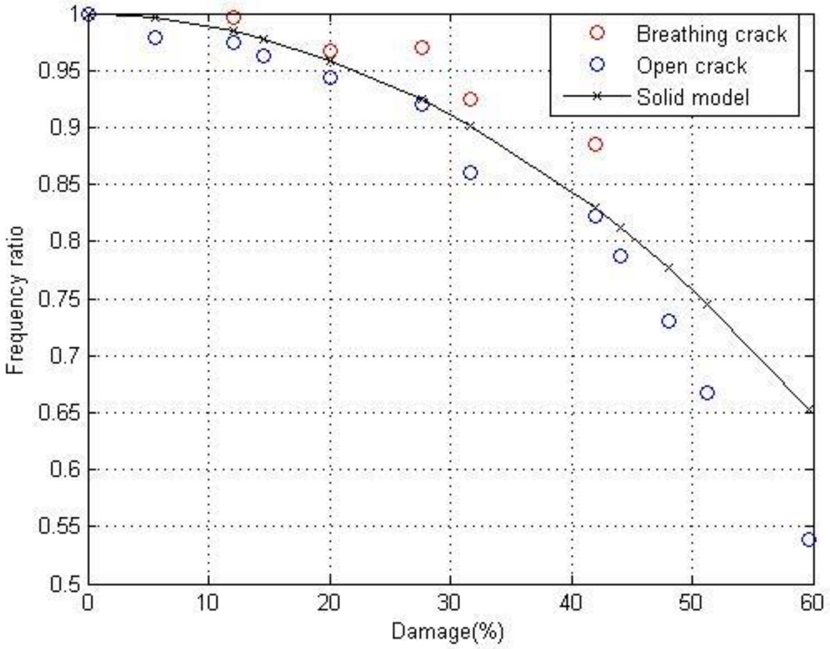


Figure 9: *Relative natural frequencies of a simply supported beam with a fatigue crack.*

4. CONCLUSIONS

The results obtained with the numerical models show that natural frequencies of beams with a notch can be accurately predicted with a solid and plate model, although the solid model is more precise. The mode shapes can also be obtained numerically in order to simulate a real experiment. A beam model can only be used with caution since the values cannot be directly related to a real situation.

Nevertheless, a real nonlinear crack may not be properly modelled with a linear finite element model, even if the thickness of the notch in the model is reduced to a null value. The stiffening effect due to the contact forces between the faces of the crack is not considered in the numerical model and lower values of natural frequencies are obtained numerically (significant errors are shown in the paper). Thus, from a damage detection point of view, the numerical response of the beam will emphasize the effect of damage and could make that the sensitivity of a damage detection methodology is artificially and improperly magnified. This effect can be avoided from an experimental point of view is a mass is added to the beam so the crack is enforced to keep open.

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